

The School Election Code

Lesson flow, teaching prompts and complete worked answers

TEAM A	TEAM B	TEAM C
10110 (base 2)	43 (base 5)	27 (base 8)

THE MISSION

Decode three vote totals written in different number bases. Compare them fairly and explain the result before the school assembly begins.

Senior 1 Mathematics | Topic 1: Number Bases

Clear steps. Real thinking. Room to try again.

Lesson at a glance

**CURRICULUM
FOCUS**

Senior 1 Mathematics - Number Bases. Suggested time: 60-75 minutes online, or two 40-minute classroom periods with the workbook.

Learning outcomes

By the end of the mission, learners should be able to:

- explain a base as a grouping and exchange rule;
- identify valid digits for a stated base;
- use place value to convert a number to base ten;
- use repeated division to convert from base ten;
- compare numbers written in different bases;
- explain a conclusion using mathematical evidence.

Materials

Student workbook, pencils, board or chart paper, and at least 20 counters, bottle tops, beans or folded ballot slips for grouping.

Key vocabulary

WORDS TO USE

base, grouping rule, bundle, exchange, digit, place value, convert, remainder, equivalent

The foundation learners need

Many learners can copy a conversion method without understanding what the base means. Begin with concrete objects. The central idea is: a base tells us when to bundle and exchange into the next place-value column.

Concrete demonstration with 13 objects

1. Place 13 counters where everyone can see them.
2. In base ten, circle or tie 10 counters together. Point to 1 bundle of ten and 3 loose ones. Write 13 (base ten).
3. Undo the bundle. In base five, make bundles of 5. Point to 2 bundles of five and 3 loose ones. Write 23 (base five).
4. Emphasise that the quantity did not change. Only the grouping rule changed.
5. Ask learners to check: $2 \times 5 + 3 = 13$.

SAY THIS

The base is the bundling rule. The digits tell us how many bundles are in each place. The small subscript labels the rule; it is not a number we multiply.

Build the place-value house

Write 243 in two place-value houses. In base ten the columns are 100, 10, 1. In base five they are 25, 5, 1. Ask why the left base-five column is 25: five groups of five make the next bundle.

Suggested lesson flow

5 min	Hook	Reveal the three coded result slips. Ask: Can we compare them? Justify they are?	Predict the result
15 min	Grouping	Bundle 13 objects first in tens, then in fives. Name the exchange like rebuild and explain.	Build exchange like
10 min	Valid digits	Connect allowed digits to the moment a full bundle is exchanged and explain errors.	Test exchange
15 min	To base ten	Compare base-ten and base-five place-value houses before place-value tables.	Complete place-value tables
10 min	From base ten	Model repeated division and reading remainders up to 100.	Convert one example with a partner.
15 min	Scenario	Ask groups to decode each team before choosing a component and defend the result.	Compare results
5 min	Exit	Return to the question: Can Team C be called 27? Explain what the base label changes.	

Questions that deepen thinking

- How do you know a number is invalid before doing any calculation?
- Why is the subscript part of the number's meaning?
- What stays the same when a number is converted?
- Which method could convince someone who disagrees with your answer?

Workbook page: valid digits

- 102 (base 2) is invalid because digit 2 is not allowed in base 2.
- 341 (base 5) is valid because every digit is from 0 to 4.
- 728 (base 8) is invalid because digit 8 is not allowed in base 8.
- Base 4 uses 0, 1, 2, 3.
- The largest digit in base 9 is 8.
- 57 (base 5) is impossible because both 5 and 7 are outside the allowed digit set 0 to 4.

Common misconception

WATCH FOR

Learners may say that base 5 uses digits 1 to 5. Return to base ten: it uses 0 to 9, not 1 to 10. Zero is always included.

To base ten

$$1101 \text{ (base 2)} = 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = 13.$$

$$243 \text{ (base 5)} = 2 \times 25 + 4 \times 5 + 3 \times 1 = 73.$$

$$36 \text{ (base 8)} = 3 \times 8 + 6 = 30.$$

Practice answers:

$$1) 10101 \text{ (base 2)} = 21.$$

$$2) 34 \text{ (base 5)} = 19.$$

$$3) 127 \text{ (base 8)} = 87.$$

$$4) 304 \text{ (base 5)} = 3 \times 25 + 0 \times 5 + 4 = 79.$$

From base ten

$$14 \text{ (base ten)} = 1110 \text{ (base 2)}.$$

$$38 \text{ (base ten)} = 123 \text{ (base 5)}.$$

$$45 \text{ (base ten)} = 55 \text{ (base 8)}.$$

SUPPORT

Let learners build groups with counters first: for base 5, exchange every five counters for one counter in the next place. Then connect the physical exchange to division and remainders.

Decode the election

Team A: $10110 \text{ (base 2)} = 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 = 22$.

Team B: $43 \text{ (base 5)} = 4 \times 5 + 3 = 23$.

Team C: $27 \text{ (base 8)} = 2 \times 8 + 7 = 23$.

Correct announcement

Teams B and C are tied with 23 votes each. Team A has 22 votes. The school should follow its tie procedure rather than announce a winner.

What a strong explanation includes

- all three conversions, not only the largest-looking digits;
- correct place values for each base;
- a direct comparison in base ten;
- a conclusion that matches the calculations.

Assessment and differentiation

Evidence of learning

Secure: learner converts accurately and explains the tie using place value.

Developing: learner identifies valid digits and converts with a place-value prompt.

Beginning: learner needs concrete grouping and support identifying place values.

Support

- Keep a place-value strip visible.
- Start with two-digit numbers.
- Let learners annotate every digit with its place value.
- Pair oral explanation with written working.

Stretch

- Ask learners to invent a fourth result equal to 23 in base 2, base 4 or base 6.
- Ask whether 132 (base 3) is valid before converting. It is valid and equals 14 in base ten.
- Compare two conversion methods and decide which is easier to check.

Exit-ticket model

The value of a number depends on its digits, their positions and the base being used. For example, 27 (base 8) means 2 groups of eight and 7 ones, so its base ten value is 23, not 27.

The Learn with Alicia approach

Let learners predict, represent, calculate and explain. A correct answer matters; a visible line of reasoning matters even more.